Calculus Warm Up #3-4

Use the fundamental theorem to find the area between f(x) and the x-axis on [0, 2]

$f(x) = 2x^2 - 3x + 2$

HW Questions, p. 267

In Exercises 1–24, evaluate the definite integral.

1. $\int_0^1 2x\,dx$

3) $\int_{-1}^{0} (x-2)\,dx$

5. $\int_{-1}^{1} (t^2 - 2)\,dt$

7. $\int_0^1 (2t-1)^2\,dt$

9. $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx$

16. $\int_{1}^{2}(5x^4+5)\,dx$

13. $\int_{-1}^{1}(\sqrt[3]{t}-2)\,dt$

15. $\int_{1}^{4}\frac{u-2}{\sqrt{u}}\,du$

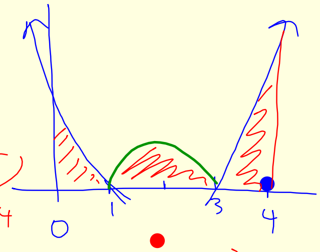
17. $\int_{0}^{1}\frac{x-\sqrt{x}}{3}\,dx$

19. $\int_{-1}^{0}(t^{1/3}-t^{2/3})\,dt$

21. $\int_{-1}^{1}|x|\,dx$

23. $\int_0^4 \left|x^2-4x+3\right| dx$

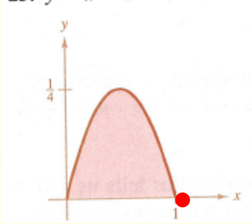
$= \int_0^1 (x^2-4x+3)dx + \int_1^3 -(x^2-4x+3)dx + \int_3^4 (x^2-4x+3)dx$



$=2\int_0^1 (\quad)dx - \int_1^3 (\quad)dx$

In Exercises 25–30, determine the area of the indicated region.

25. $y = x - x^2$



$A = \int_0^1 (x-x^2)dx$

$\Big[\qquad\Big]_0^1$

5.4 -The Fundamental Theorem of Calculus ✓

-The Mean Value Theorem for Integrals

-Average value of a function on an interval

-The Second Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus

Essentially: Differentiation and Definite Integration are inverse operations.

If f is continuous on [a, b], and F'(x) = f(x)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

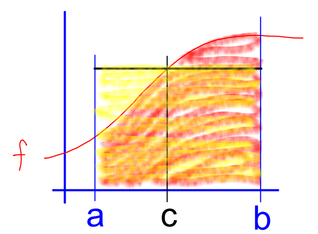
The Mean Value Theorem for Integrals

Somewhere on [a, b] there is an x-value, c, where the area of the rectangle:

A = (Height)(Width)

A = ($f(c)$)(b - a)

Is exactly equal to the area under the curve.



Mean Value Rectangle

MVT for Integrals is an Existence Theorem

If f is continuous on [a, b], then there exists an x-value, c, in the interval such that:

$$(f(c))(b - a) = \int_a^b f(x)\,dx$$

Area of yellow rectangle = Area under curve

MVT for Integrals:

$$(f(c))(b - a) = \int_a^b f(x)\,dx$$

The value of $f(c)$, given in the Mean Value Theorem for Integrals, is called the **average value** of f on the interval $[a, b]$.

Definition of the Average Value (outcome) of a Function on an interval.

If f(x) is integrable on [a, b], then
f(c) = average value

$$f(c) = \frac{1}{b - a}\int_a^b f(x)\,dx$$

Find the average value of f(x) on [1, 4]

$$f(x) = 3x^2 - 2x$$

$$f(c) = \frac{1}{b - a}\int_a^b f(x)\,dx$$

$$f(c) = \frac{1}{3}\int_1^3 (3x^2 - 2x)\,dx$$

$$\frac{1}{3}\left[\frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{2}x^2}{\cancel{2}}\right]_1^3$$

$$\frac{1}{3}\left[3^3 - 3^2 - (1^3 - 1^2)\right]$$

Developing

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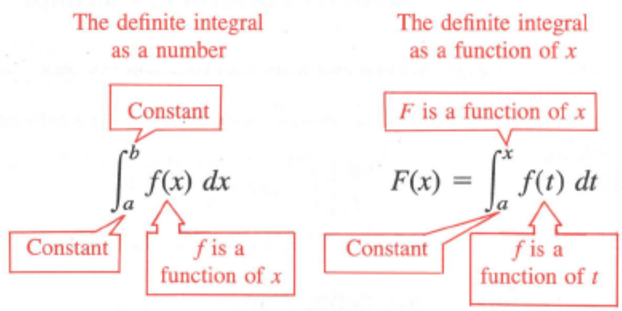
The Second Fundamental Theorem of Calculus

When we defined the definite integral of f on the interval $[a, b]$, we used the constant b as the upper limit of integration and x as the variable of integration. We now look at a slightly different situation in which the variable x is used as the upper limit of integration. To avoid the confusion of using x in two different ways, we temporarily switch to using t as the variable of integration. (Remember that the definite integral is *not* a function of its variable of integration. Moreover, any variable can be used.)



The definite Integral as a function:

$$F(x) = \int_a^x f(t)\,dt$$

Evaluate:

$$F(x) = \int_0^x (3 - 3t^2)\,dt$$

$$= \left[3t - t^3\right]_0^x$$

$$F(x) = 3x - x^3 - (\ 0\ )$$

How can we find f(x)? $= \frac{d}{dx}\left[F(x)\right] = f(x)$

$$\frac{d}{dx}(3x - x^3) = 3 - 3x^2$$

The Second Fundamental Theorem of Calculus

Fomalizing the idea that differentiation and integration are inverse operations.

$$f(x) = \frac{d}{dx}\int_a^x f(t)\,dt$$

EXAMPLE 6 Applying the Second Fundamental Theorem of Calculus

Evaluate

$$\frac{d}{dx}\int_0^x \sqrt{t^2+1}\,dt. = \sqrt{x^2+1}$$

HW: p. 267 # 27 - 51 odd

(skip 41)

HW Quiz Friday:

pgs. 238, 248, 249, 258, and

trapezoids p. 287 / midpoints p. 250

THEOREM 5.10 PRESERVATION OF INEQUALITY

1. If f is integrable and nonnegative on the closed interval $[a, b]$, then

$$0 \leq \int_a^b f(x)\, dx.$$

2. If f and g are integrable on the closed interval $[a, b]$ and $f(x) \leq g(x)$ for every x in $[a, b]$, then

$$\int_a^b f(x)\, dx \leq \int_a^b g(x)\, dx.$$

In Exercises 31–34, find the area of the region bounded by the graphs of the given equations.

33. $y = x^3 + x,\ x = 2,\ y = 0$

In Exercises 35–38, find the values of c guaranteed by the *Mean Value Theorem for Integrals* for the given function over the specified interval.

	Function	*Interval*
37.	$f(x) = -x^2 + 4x$	$[0, 3]$

In Exercises 39–42, sketch the graph of the given function over the specified interval. Find the average value of the function over the interval and all values of x where the function equals its average value.

Function	*Interval*
41. $f(x) = x - 2\sqrt{x}$	$[0, 4]$

In Exercises 43–48, (a) integrate to find F as a function of x and (b) demonstrate the Second Fundamental Theorem of Calculus by differentiating the result of part (a).

45. $F(x) = \int_8^x \sqrt[3]{t}\, dt$

In Exercises 49–52, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

49. $F(x) = \int_{-2}^x (t^2 - 2t + 5)\, dt$

53. The volume V in liters of air in the lungs during a 5-second respiratory cycle is approximated by the model

$$V = 0.1729t + 0.1522t^2 - 0.0374t^3$$

where t is the time in seconds. Approximate the average volume of air in the lungs during one cycle.

In Exercises 25–30, determine the area of the indicated region.

29. $y = \sqrt[3]{2x}$